A BRIEF HISTORY OF MATHEMATICS AND PHYSICS

RITIK JAIN

INTRODUCTION

In the history of science, there has been perhaps no greater miracle than the unreasonable effectiveness of mathematics in describing our universe.[1] Indeed, since Newton's discovery of calculus-based mechanics, seemingly abstract mathematics such as group theory and Riemannian geometry has produced a sequence of fundamental breakthroughs in physics, each more spectacular than the last. Conversely, problems in physics have time and again motivated the creation of entirely new fields of mathematics, for instance Fourier analysis and partial differential equations.

In this note, we present a timeline of these events, emphasizing key results, central figures, and the recurring themes that link mathematics and physics. We place a special emphasis on the pervasive role of symmetry in the two disciplines, a deep concept whose universal significance has only become more apparent over time.

While we broadly survey the most fundamental developments, some important context and discoveries are excluded in the interest of brevity. For a fuller treatment, especially concerning the pre-1600 period, see [2], [3]. The colorful lives of the mathematicians behind the developments are equally worth knowing: see [4] for an enthralling account.

1600-1700: FOUNDATIONS

This century was a particularly fertile period for mathematics and physics, giving rise to several entirely new subfields. In mathematics, Descartes developed elementary algebra and introduced many modern symbolic notations (e.g. the use of x to denote unknowns). He also introduced the coordinate plane, bringing algebraic rigor to geometry and the intuition of geometry to algebra. In physics, Kepler used Descartes' coordinate geometry to formulate his laws of planetary motion, additionally drawing connections to the study of conic sections. Concurrently, Galileo popularized Copernicus' heliocentric model and performed several influential experiments on gravitation.

This work culminated in the landmark development of calculus by Newton and Leibniz. With this new framework, Newton simultaneously transformed almost every area of

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[1] For a captivating discussion of this interplay, see Nobel laureate Eugene Wigner's influential essay [1].

physics and proved several original results in pure mathematics, marking him as one of the greatest scientific minds in human history. By the close of the century, he shattered the philosophical physics of Aristotle that reigned supreme for nearly 2,000 years, thereby setting the stage for modern physics.

Timeline:

- Descartes introduces analytic geometry and Cartesian coordinates.
- Napier and Bürgi introduce logarithms, simplifying astronomical calculations.
- Kepler states his laws of planetary motion.
- Galileo develops a version of relativity, which states that the laws of motion are the same in any inertial frame of reference, e.g. in a moving airplane or a room.
- Fermat rediscovers the work of Diophantus in number theory and proves several new results, revitalizing interest in the subject.
- Fermat and Pascal study games of chance, initiating probability theory.
- Newton and Leibniz independently discover calculus.
- Newton publishes *Principia Mathematica*, in which he mathematically derives the mass and shape of the Sun and Earth, his laws of motion, and the law of universal gravitation. In doing so, Newton achieves the first great unification of physics.
- Newton uses coordinate geometry to study Diophantine equations, states Bézout's theorem, and classifies cubic curves, thus initiating classical algebraic geometry.
- Johann Bernoulli invents the calculus of variations, the study of optimizing integral functions. With his brother, Jakob, he solves the brachistochrone problem.

1700-1800: The Age of Discovery

The period 1700–1800 was characterized by the rapid expansion and application of the ideas introduced in the previous century. In spite of the extreme political tumult of the century, especially due to the French Revolution, a mathematical community formed in Europe, with mathematicians such as Lagrange, Laplace, Fourier, and Jacobi making fundamental contributions across areas as diverse as classical mechanics, number theory, and engineering. This progress was largely led by the development of calculus, which was shaped into a powerful and versatile tool.

Euler emerged far and away as the premier mathematician of this era, publishing hundreds of pages of research per year even after he was afflicted with blindness in his old age. His work led to breakthroughs in innumerable subjects: for instance, he introduced the notion of a mathematical function, invented the field of graph theory by solving the Seven Bridges of Königsberg problem, and defined the Euler characteristic of a polyhedron in an early precursor to algebraic topology. He also made fundamental contributions to applied mathematics, writing several volumes on optics, marine engineering, load-bearing beams, and fluid dynamics. These texts were largely phrased in terms of partial differential equations, which he himself played a central role in developing. Overall, Euler's contributions set the tone of the century, not only leading to huge advancements in mathematics and physics, but in Europe's technological prowess as a whole.

Timeline:

- Euler and others put calculus to good use in solving various problems in mechanics, fluid dynamics, astronomy, and engineering.
- Euler introduces analytic methods to number theory. He shows that the sum of the reciprocals of the primes diverges, solves the Basel problem, and proves the Euler product formula.
- Differential equations emerge as a core tool in mathematics and physics; Euler solves the general homogeneous linear ODE with constant coefficients.
- Lagrange reformulates classical mechanics without coordinates using the calculus of variations and proves the divergence theorem.
- Laplace makes groundbreaking advances in PDEs and applies these methods to physics and celestial mechanics.
- Fourier analysis is invented, which studies functions as superpositions of trigonometric functions.
- Legendre conjectures the prime number theorem, an asymptotic growth rate of the prime numbers.
- Gauss proves the fundamental theorem of algebra; the complex plane is invented.

1800-1900: Unification and Abstraction

The 1800s were a time of great change in modern mathematics and physics. In mathematics, the ad-hoc progress of the previous centuries was organized and greatly built upon, leading to the birth of modern algebra, geometry, and analysis among other fields. The connections between all of these fields were also systematically investigated for the first time, and in particular the universal significance of symmetries was understood. The fundamental contributions of Gauss and Riemann to geometry, number theory, and analysis were particularly influential.

These developments had a profound impact on theoretical physics. For one, Lagrange's classical mechanics was reinterpreted geometrically by Hamilton and Poincaré. Informed by Klein's *Erlangen* program in pure geometry, this led them to develop symmetry principles for physical systems. Concurrently, Maxwell derived his fundamental equations of electromagnetism using PDEs, laying the groundwork for Einstein's theory of special relativity.

Timeline:

- Gauss writes *Disquisitiones Arithmeticae* at 21, a groundbreaking work which essentially contains the whole of elementary number theory as it is known today.
- Jacobi and Abel initiate the study of elliptic functions and elliptic integrals.
- Galois discovers group theory by his study of symmetries of polynomial roots. These findings would later develop into what is now known as Galois theory.
- Gauss and others lay the foundations of differential geometry; non-Euclidean geometry is discovered (Lobachevsky, Bolyai).
- Cauchy formally defines limits and continuity, introducing rigor to analysis; this is later systematized by Weierstrass.
- Cauchy proves several fundamental results in complex analysis, establishing it as an important field in its own right.
- Riemann introduces geometric methods to complex analysis (Riemann surfaces), and develops Riemannian geometry, advancing Gauss' work. He develops the concept of a manifold, which is, informally, a smooth structure which is "locally flat", e.g. a sphere, plane, or Euclidean space more generally.
- Motivated by outstanding questions such as Fermat's Last Theorem, Dedekind, Kronecker, and Kummer invent the field of algebraic number theory.
- Cayley and others formalize linear algebra and group theory. Frobenius develops representation theory, a method of studying groups using linear algebra.
- Analytic number theory is developed by Dirichlet and Riemann, who, expanding on Euler's work, develop relations between the prime numbers and certain power series. Dirichlet proves the infinitude of primes in an arithmetic progression.
- Maxwell formulates the equations of electromagnetism; the symmetries and geometry of these equations are discovered by Poincaré and Lorentz.
- Boltzmann, Gibbs, and Maxwell found the field of statistical mechanics, which studies ensembles of particles using probabilistic methods.
- Klein introduces his *Erlangen* program, which aims to classify geometric spaces by their symmetry groups; this perspective shapes future developments in geometry.
- Cantor creates set theory and extends the notion of cardinality to infinite sets.
- The prime number theorem is proven using Riemann's zeta function.
- Lie develops the theory of smooth groups (Lie groups), extending Galois' symmetry methods to analysis and geometry. The significance of his work in particle physics and geometry would only be realized some 30 years after his death.
- Poincaré founds algebraic topology, introducing homotopy and homology, which measure "obstructions" on surfaces using group theory.

1900-2000: A Creative Explosion

The 20th century was, with little doubt, one of the most extraordinary periods in the history of mathematics and physics, yielding a dizzying number of fundamental results across vastly different areas. This period was essentially split into two halves. From 1900-1950, theory continued to dominate, with groundbreaking progress in both mathematics and physics. During this time, the two fields also continued to progress in lockstep, with new methods in geometry, functional analysis, and abstract algebra feeding directly into innovations in physics.

From 1950-2000, several changes occurred in both mathematics and physics. While mathematics turned inwards and developed to a level of extreme abstraction, physics became largely more experimental, focusing on testing and absorbing the large amount of theory developed in the prior decades. Efforts on both sides reached a local peak in the 1970s, with the large-scale completion of Grothendieck's program in algebraic geometry and the completion of the Standard Model in particle physics.

From the 1980s and onwards, the attention shifted in mathematics to solving concrete problems, with a highlight being Wiles' proof of Fermat's Last Theorem. In physics, a slew of experimental results were coupled with a resurgence in string theory, leading to the discovery of deep links between geometry and quantum field theory.

Due to the sheer amount of new results, we will restrict ourselves to discussing mostly pure mathematics and theoretical physics, split by decade. However, this century was also transformative for applied mathematics and experimental physics. Indeed, progress in both domains led to such feats of human endeavor as the invention of modern computers, the birth of artificial intelligence, and the Moon landing.

1900–1910.

- Hilbert poses his 23 problems, helping to set the research agenda for 20th century mathematics.
- Einstein proves the mass-energy equivalence formula $E = mc^2$.
- Lebesgue discovers measure theory and defines the Lebesgue integral, generalizing Riemann's construction.
- Hausdorff gives the modern definition of a topological space.
- Peano and Russell further develop the foundations of mathematics. Russell shows that set theory is not a consistent foundation (Russell's paradox).
- Hilbert, Fredholm, and Riesz develop functional analysis.

1910–1920.

- Continuing Poincaré's work, Weyl advances the role of geometry and symmetry in physics. He introduces gauge theory, the study of systems which are invariant under smooth symmetries, and provides the first intrinsic definition of a manifold.
- Einstein formulates the theory of general relativity and proves his namesake field equations, which describe space and time as a curved 4-dimensional manifold

distorted by gravity. This spectacular application of Riemannian geometry is regarded as one of humanity's greatest intellectual achievements.
- Noether proves that every conservation law in physics corresponds to a symmetry of the universe, deeply linking abstract algebra and physics (Noether's theorem).
- Hardy and Ramanujan collaborate on a series of papers. Led by Ramanujan's otherworldly insight, they conjecture and prove several seemingly miraculous formulas involving arithmetic functions and modular forms. Their contributions go on to guide research in number theory.

1920–1930.

- Emmy Noether formulates modern commutative algebra and ring theory.
- The Zermelo–Fraenkel axioms (ZFC) are established, putting mathematics on a solid axiomatic foundation.
- Weyl develops the representation theory of Lie groups and Lie algebras with applications to physics.
- Bohr establishes his namesake institute in Copenhagen, which would become the birthplace of modern quantum mechanics.
- Among others, Max Born, Heisenberg, and Schrödinger develop the foundations of quantum mechanics. This work is mathematically formalized in terms of functional analysis and Hilbert spaces by Dirac, Hilbert, and von Neumann.
- Dirac derives his namesake equation, proving that quantum mechanics is mathematically consistent with Einstein's special relativity and predicting the existence of antimatter.
- von Neumann publishes *On the Theory of Games*, establishing game theory.

1930–1940.

- Banach develops abstract functional analysis.
- Zariski introduces commutative algebra and topology to algebraic geometry.
- Gödel proves his incompleteness theorems, showing that any reasonable axiomatic formulation of mathematics contains true yet unprovable statements.
- Kolmogorov publishes *Foundations of Modern Probability*, setting probability on a rigorous, measure-theoretic basis.
- Church and Turing respectively define λ-calculus and the Turing machine, thus creating modern computer science.
- Oppenheimer uses Einstein's field equations to predict the existence of black holes.
- Wigner introduces representation theory to particle physics.

1940–1950.

- Mac Lane and Eilenberg develop category theory.

- Algebraic topology is developed into its modern categorical form by Cartan, Eilenberg, and Steenrod.
- Weil publishes *Foundations of Algebraic Geometry*, a first step towards Grothendieck's generalization of algebraic geometry.
- Feynman, Schwinger, and Tomonaga develop quantum field theory (QFT) and the technique of renormalization.
- Nuclear and particle physics see rapid progress; the atom bomb is created.
- Hamming and Shannon pioneer information theory.

1950–1960.

- Iwasawa and Tate introduce methods of harmonic analysis to number theory.
- Serre makes foundational contributions to algebraic topology and proves the finiteness of the homotopy groups of spheres. He introduces Grothendieck to algebraic topology and algebraic geometry.
- Serre publishes his influential "GAGA" paper (*Géométrie Algébrique et Géométrie Analytique*), developing sheaf theory and homological algebra.
- Milnor develops differential topology, discovering the existence of exotic spheres.
- Yang and Mills reformulate particle physics in terms of representations of compact Lie groups, forming the basis of the Standard Model.
- Over a 5-year period, Grothendieck revolutionizes algebraic geometry, topology, category theory, and homological algebra. He defines schemes, which provide the correct generalization of algebraic geometry to arbitrary commutative rings.

1960–1970.

- Grothendieck publishes *Éléments de Géométrie Algébrique* (EGA) and related notes which total over 5,000 pages, disseminating his ideas to a broader mathematical audience.
- Atiyah, Hirzebruch, and others create K-theory, a vast generalization of (co)homology.
- Smale proves the Poincaré conjecture in dimensions greater than 5.
- Stochastic processes and martingale theory are developed using Kolmogorov's measure-theoretic probability, laying the groundwork for modern statistics.
- Langlands proposes a set of far-reaching conjectures connecting Fourier analysis, representation theory, and number theory, inaugurating the Langlands program.
- Atiyah and Singer prove the index theorem, a deep result in geometric analysis which contains several important theorems as special cases.

1970–1980.

- Golfand and Likhtman propose supersymmetry, a symmetry between bosons and fermions, as a possible solution to the hierarchy problem in particle physics.

- Deligne proves the Riemann hypothesis for curves over finite fields, a stunning application of Grothendieck's scheme-theoretic algebraic geometry.
- Bekenstein and Hawking investigate the thermodynamics of black holes; Hawking radiation is discovered.
- Particle physics undergoes a renaissance, culminating in the completion of the Standard Model by Weinberg, 't Hooft, and Salam among others.

1980–1990.

- Quantum computing is proposed by Feynman.
- Faltings proves Mordell's conjecture, which shows the finitude of the number of rational points on certain curves defined over a number field. This result implies a weak form of Fermat's Last Theorem (FLT).
- In another step towards the proof of FLT, Ribet resolves the ε-conjecture.
- Donaldson uses Yang-Mills theory from physics to prove the existence of exotic smooth structures on 4-dimensional space.
- Thurston proposes his geometrization program; surgery theory is developed as a method of analytically decomposing manifolds into simpler pieces.
- String theory is proposed as a possible theoretical framework for the Standard Model, sparking the first superstring revolution.

1990–2000.

- Wiles proves the Taniyama-Shimura conjecture for semistable elliptic curves, hence resolving Fermat's Last Theorem.
- The full Taniyama-Shimura conjecture is proven by Conrad, Diamond, and Taylor, all former students of Wiles.
- The second superstring revolution occurs, culminating in Witten's unification of the 5 possible string theories as limits of a single theory (M-theory).
- Mirror symmetry is developed, linking algebraic geometry to string theory.
- Donaldson, Witten, and others develop topological quantum field theory, finding a deep relationship between the theory of knots, 3-manifolds, and quantum fields.
- Maldacena proposes the AdS/CFT correspondence, connecting string-theoretic quantum gravity and quantum field theory.

2000-Present: Resolution

Over the past 25 years, progress in mathematics and theoretical physics has largely focused on collecting and applying the whirlwind of new theory produced by the previous century. At the same time, both fields have become highly technical, with recent progress being comparatively incremental and largely driven by collaborative efforts between many specialists. In this section, we will collect some notable new results.

- Perelman resolves the Poincaré conjecture using geometric analysis, in particular surgery theory and Ricci flow.
- Green and Tao prove that the prime numbers contain arbitrarily long arithmetic progressions.
- In a joint effort by over a hundred mathematicians, a full classification of finite simple groups is achieved.
- Progress continues on the Langlands program; Ngô proves the fundamental lemma, and a team led by Gaitsgory resolves the geometric Langlands conjecture.
- The Higgs boson is observed at CERN, completing the experimental confirmation of the Standard Model.
- Hales produces a formal proof of the Kepler conjecture regarding sphere-packing in Isabelle, an automated theorem prover.
- Gravitational waves are observed for the first time at LIGO.

Outlook

Standing in the shadow of the gigantic achievements of our predecessors, one might feel as if theoretical advancement in mathematics and physics has reached its limit. While many important problems remain open in both fields, progress has become mostly incremental; in mathematics, new proofs of important results often fill thousands of pages of technical material, and in physics, once-promising theoretical advances such as string theory have become increasingly arcane and divorced from physical reality.

Such periods, however, are far from unprecedented. A similar time was the late 19th century, when Newtonian mechanics reached its full maturity, and many eminent physicists felt as though their field was near completion. For instance, an 1899 quote from Nobel Laureate Albert Michelson reads:

> The more important fundamental laws and facts of physical science have all been discovered, and these are now so firmly established that the possibility of their ever being supplanted in consequence of new discoveries is exceedingly remote.

Although Michelson's statement may seem short-sighted today, it should be noted that he had good reason to believe this to be true. Indeed, Newtonian mechanics was able to account for virtually all observable phenomena, and the few experiments that contradicted it (including, ironically, the Michelson-Morley experiment) appeared to be simple edge cases that could be accounted for by mild adjustments.

Today, we are confronted with a similar situation. While there are known edge cases at which today's physics breaks down, e.g. the center of a black hole, the predictive power of our theories is so strong that after a century of effort, we have yet to observe any deviation from them! However, if we are to take any lesson from the late 19th century, it is that this apparent completeness may indicate that physics is ripe for renewal, and these very edge cases likely point toward a deeper unifying theory.

But what might progress in the 21st century look like? For one, there is little doubt that it will be more collaborative than the past, as multi-author papers continue to rise in popularity. Moreover, AI and machine learning will likely play an increasingly important role; cf. recent work applying deep learning to astrophysics [5] and mathematical discovery [6]. Closely related is the emergence of automated theorem provers such as Lean, which are formal languages in which mathematics can be encoded and programmatically verified. One exciting related area is *autoformalization*, which aims to integrate such systems with large language models: see for example [7].

On the theoretical side, there are also many intriguing research directions. In physics, work on string theory has revealed striking connections between quantum field theory, number theory, and algebraic geometry: see for instance [8]. While these connections remain mysterious, our experience leads us to believe their exquisite mathematical beauty is a strong indicator that arithmetic and algebro-geometric objects will play a large role in any future unifying theory. In pure mathematics, efforts to simplify complicated technical frameworks have led to the discovery of new unifying structures: most notably, Scholze's theory of perfectoid spaces [9] has provided a bridge between p-adic geometry and the arithmetic of function fields over $\mathbb{F}_p$. This connection has led to substantial progress in the Langlands program, which itself has emerged as a central theme in mathematics.

Thus, looking ahead, we have little reason to despair. Although theoretical progress may have slowed and the open problems that remain appear formidably difficult, recent computational and conceptual advances offer reason for optimism. And, if history is any guide, the slowdown we are experiencing today is not a sign of exhaustion, but a harbinger of the next great revolution in mathematics and physics.

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